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03 Kinematics (Motion in a Straight Line & Plane)

Motion in a Straight Line

Introduction of Kinematics

Rest

If position does not change with time, then it is at rest.

Motion

If position of particle changes with time, then it is called in motion.

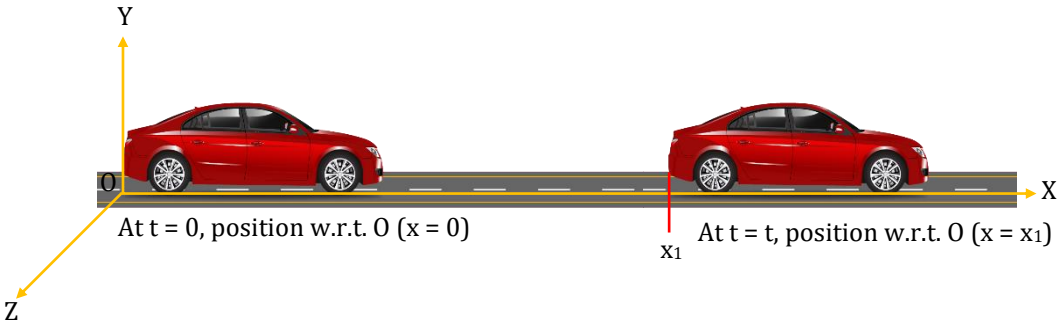

Illustration 1:

Identify examples of rest and motion : Walking, Sleeping, dancing, playing, standing, statue, ball throwing, cycling.

Solution:

Example of motion : - Walking, dancing, playing, ball throwing and cycling.

Example of rest : - sleeping, standing, statue

Frame of Reference

1. It is the reference with respect to which position or motion of particle is defined.
2. Motion and Rest are relative terms which depends on Frame of reference.
3. If not specified, we consider ground as reference frame.

Conclusion (With Respect to a Reference point)

1. If position of particle changes with time, then it is in Motion.
2. If position of particle does not change with time, then it is in Rest.

Illustration 2:

Give an example where object is at rest and also in motion at the same time

Solution:

Whether a object is in rest or motion depends upon the reference frame by which you are observing, Different reference frame may give different answers. Example: If you are travelling in a train and a woman is sitting near you. you will see the woman near you as stationary but if someone is observing the woman from outside the train he will see as if the woman is moving with the speed of train. Hence the two different reference point has given two results.

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Types of Motion

1-D Motion:

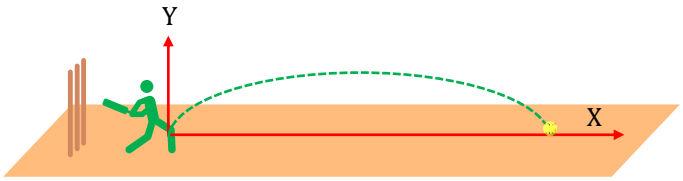
If position changes with time with respect to frame of reference along a straight line, motion is one dimensional (1-D) or straight-line motion or rectilinear motion.

Examples: Motion of Train Along Straight Track , A Ball Projected Upward.

2-D Motion:

If position changes in a plane with time with respect to frame of reference then motion is 2-D.

Examples: Projectile Motion



Examples: Circular Motion

3-D Motion:

If position changes in space with time with respect to frame of reference, motion is 3-D or motion in space.

Examples: Motions of Kite

Illustration 3:

Short following examples according to type of motion.

(1) Motion of an aeroplane
(2) Earth revolving around the sun
(3) Motion of the wheels of moving train
(4) Train running on a straight track.
(5) Lizard moving on the wall
(6) A sprinter running a 100 m race on straight track.
(7) A ball dropped from a building
(8) An out swinging delivery in cricket

Solution:

1D: - Train running on a straight track, A sprinter running a 100 m race on straight track, A ball dropped from a building.

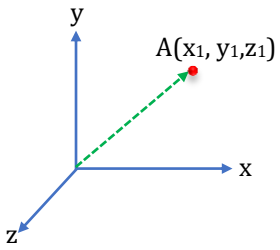
2D: - Earth revolving around the sun, Motion of the wheels of moving train, Lizard moving on the wall.

3D: - Motion of an aeroplane, An out swinging delivery in cricket.

Distance, Displacement and Position Vector

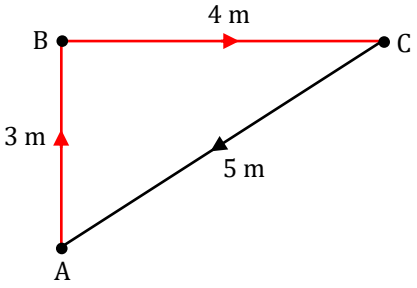
Position Vector

- It is used to specify the position of a certain particle.
- A vector that symbolizes the position of any given point with respect to any reference point like the origin.
- **Direction:** Always points from the Reference point (origin) of that vector towards a given point.



Distance

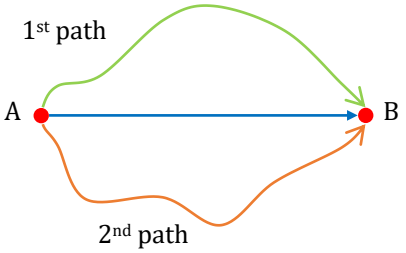
Actual length of the path traversed by the particle between initial and final position is called distance.



Distance travelled by man through path ABCA = 3 + 4 + 5 = 12m

Key Point:

- Distance always depends on path which is followed by the particle. Does not depend on initial and final position of the particle.

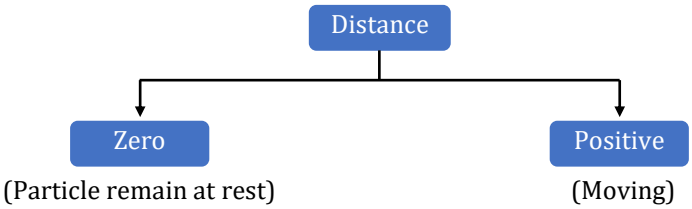


$(\text{Distance})_1 \neq (\text{Distance})_2$

- Infinite distances are possible between two fixed points because infinite paths are possible between two fixed points.
- It is a Scalar Quantity.
- Unit

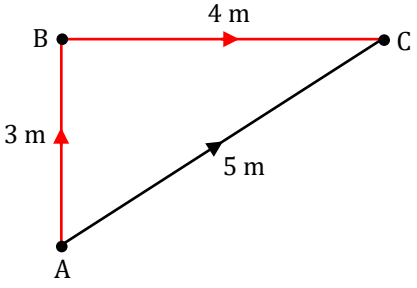
  S.I. → Metre (m)

  C.G.S. → Centimetre (cm)
- Dimension: $[M^0L^1T^0]$
- Distance cannot be negative.



Displacement

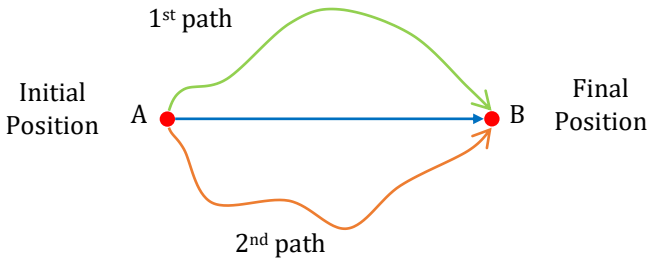
Shortest distance between initial & final position of the particle is called displacement.



Displacement of man when moving from A to B and B to C = 5m

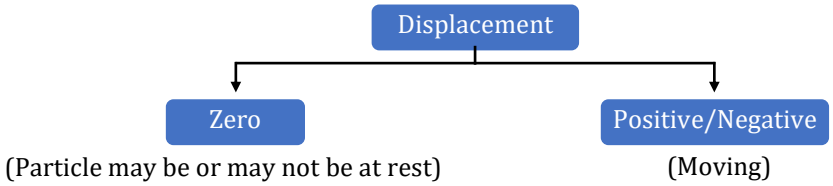
Key Point:

- Displacement is a vector quantity, and its direction is always from initial position to final position.
- Displacement does not depend on path followed by particle. It depends only on initial & final position of the particle.



$(\text{Displacement})_1 = (\text{Displacement})_2$

- Only single value of displacement is possible between two fixed points.
- Displacement may be positive, negative or zero.
- If motion is in straight line without change in direction then distance = |displacement| = magnitude of displacement.
- Magnitude of displacement may be equal or less than distance but never greater than distance. i.e., distance $\geq$ |displacement|



Displacement in Vector Form

Initial position vector $\left(\overrightarrow{OA}\right)$

$\vec{r}_1 = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$

Final position vector $\left(\overrightarrow{OB}\right)$

$\vec{r}_2 = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$

Displacement vector $\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$

$\Delta\vec{r} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$

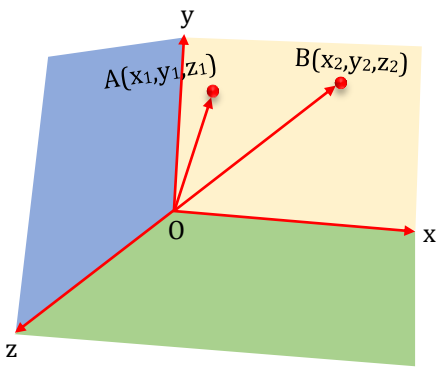


Illustration 4:

A particle starts from point A (1, –1, 0) m and reaches at point B (5, 3, –2) m, reference point is origin. Find:

(i) Initial position vector (ii) Final position vector

(iii) Displacement vector (iv) Length of shortest path

Solution:

(i) Initial position vector $\left(\overrightarrow{OA}\right) \Rightarrow \overrightarrow{OA} = \hat{i} - \hat{j}$

(ii) Final position vector $\overrightarrow{OB} \Rightarrow \overrightarrow{OB} = 5\hat{i} + 3\hat{j} - 2\hat{k}$

(iii) Displacement vector $\Delta\vec{r} = \vec{r}_2 - \vec{r}_1 = \overrightarrow{OB} - \overrightarrow{OA} = (5-1)\hat{i} + (3-(-1))\hat{j} + (-2-0)\hat{k} = \left(4\hat{i} + 4\hat{j} - 2\hat{k}\right)$

(iv) Length of shortest path $|\Delta\vec{r}| = \sqrt{4^2 + 4^2 + 4} = 6\text{m}$

Illustration 5:

A particle starts from the origin, goes along the X-axis upto the point (20m, 0) and then returns along the same line to the point (–20m, 0). Find the distance and displacement of the particle during the trip.

Solution:

Distance = |OA| + |AC| = 20 + 40 = 60m

Displacement = OA + AC = $20\hat{i}+(-40\hat{i})=(-20\hat{i})$m

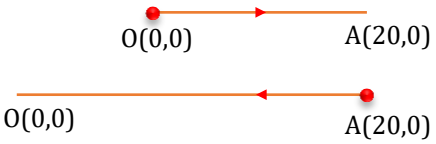
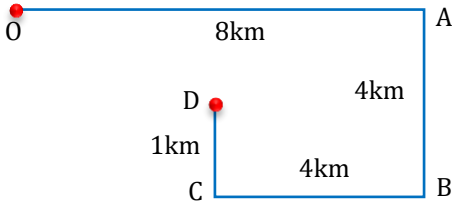
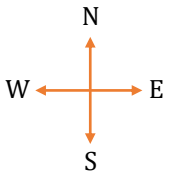


Illustration 6:

A car moves from O to D along the path OABCD shown in fig. What is distance travelled and its net displacement?





Solution:

Distance $=\left|\vec{OA}\right|+\left|\vec{AB}\right|+\left|\vec{BC}\right|+\left|\vec{CD}\right|$ = 8 + 4 + 4 +1 = 17 km

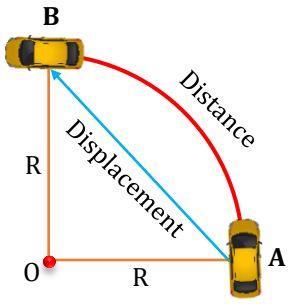
Displacement $=\vec{OA}+\vec{AB}+\vec{BC}+\vec{CD}=8\hat{i}+(-4\hat{j})+(-4\hat{i})+\hat{j}=4\hat{i}-3\hat{j} \Rightarrow |\text{displacement}|=\sqrt{(4)^2+(3)^2}=5$

So, Displacement = 5km, 37° S of E

Distance and Displacement in circular path

A car starts moving from point A and reaches at point B along a circular path then find out distance and displacement in following cases.

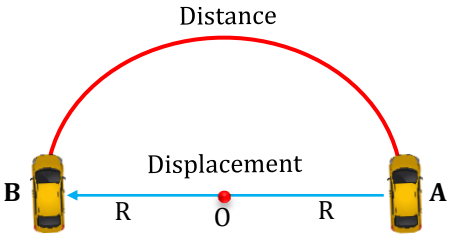
(1) One-fourth of the complete circle



Distance = $\frac{\pi R}{2}$

Displacement = $\sqrt{2}R$

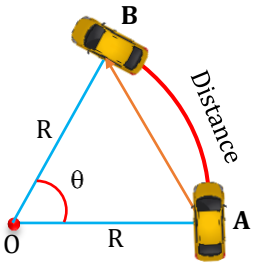
(2) Half of the complete circle



Distance =πR

Displacement = 2R

(3) At any arbitrary arc



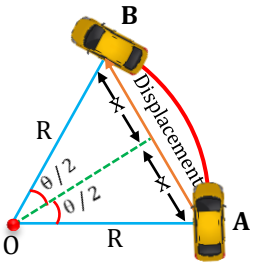


Illustration 7:

A cyclist moving on a circular track, completes one revolution in 10 sec. Then find out its displacement after 1 minute 5 s (radius = 1m)

Solution:

In 1 minute, 5s number of revolutions = 6.5

$\therefore$ Displacement = 2r = 2m

Displacement = 2x $\Rightarrow \frac{x}{R} = \sin\frac{\theta}{2}$

$\therefore$ Displacement $= 2R\sin\frac{\theta}{2}$

Useful Direction Conventions

Sense of Direction in terms of base vectors.

East : $\hat{i}$

West : $-\hat{i}$

North : $\hat{j}$

South : $-\hat{j}$

Up : $\hat{k}$

Down : $-\hat{k}$

Sense of Direction in terms of base vectors.

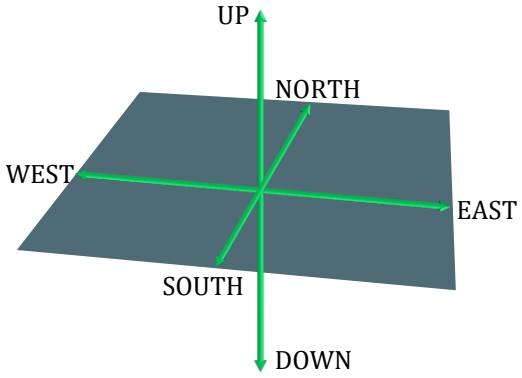


Illustration 8:

A person moves 30 m due west, then 20 m due south then 20 m due east. Find out displacement.

Solution:

$\vec{d}_1 = -30\hat{i}, \vec{d}_2 = -20\hat{j}, \vec{d}_3 = 20\hat{i}$

$\vec{d} = \vec{d}_1 + \vec{d}_2 + \vec{d}_3 = (-10\hat{i} - 20\hat{j})m$

$|\vec{d}| = 10\sqrt{5}m$

$\tan\theta = 2 \Rightarrow \theta = \tan^{-1}(2)$

$\vec{d} = 10\sqrt{5}m, \tan^{-1}(2)$S of W or $\vec{d} = 10\sqrt{5}m.$ $\tan^{-1}(2)$ (w to s)

Illustration 9:

A Body moves 6 m north 8 m east and 10m vertically upwards, what is its resultant displacement from initial position.

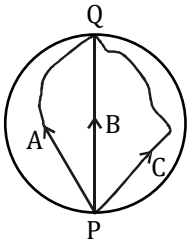
Solution:

$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ $\therefore r = \sqrt{x^2 + y^2 + z^2} = \sqrt{6^2 + 8^2 + 10^2} = 10\sqrt{2}$ m

BEGINNER'S BOX-1

1. A particle moves on a circular path of radius 'r', It completes one revolution in 40 s. Calculate distance and displacement in 2 min 20 s.

2. Three girls skating on a circular ice ground of radius 200 m start from a point P on the edge of the ground and reach a point Q diametrically opposite to P following different paths as shown in figure. What is the magnitude of the displacement for each? For which girl is this equal to the actual length of path skate?



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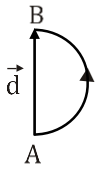
3. A man moves 4 m along east direction, then 3m along north direction, after that he climbs up a pole to a height 12m. Find the distance covered by him and his displacement.

4. A person moves on a semicircular track of radius 40 m. If he starts at one end of the track and reaches the other end, find the distance covered and magnitude of displacement of the person.



5. A man has to go 50m due north, 40m due east and 20m due south to reach a cafe from his home.

(A) What distance he has to walk to reach the cafe?

(B) What is his displacement from his home to the cafe?

Speed

The Rate at which distance is covered with respect to time is called speed.

Scalar Quantity

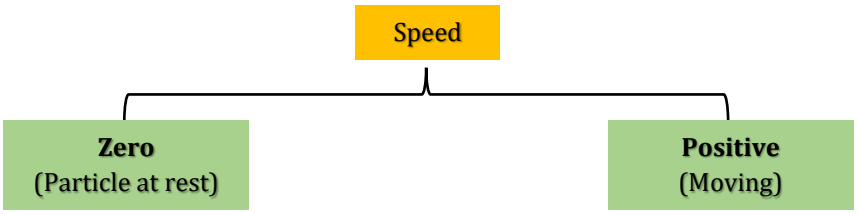
Unit : S.I. → Metre per second (m/s)

C.G.S. →Centimetre per second (cm/s)

Conversion : km/hr $\underset{\times\frac{18}{5}}{\overset{\times\frac{5}{18}}{\rightleftharpoons}}$ m/s

Dimension : $[M^0L^1T^{-1}]$

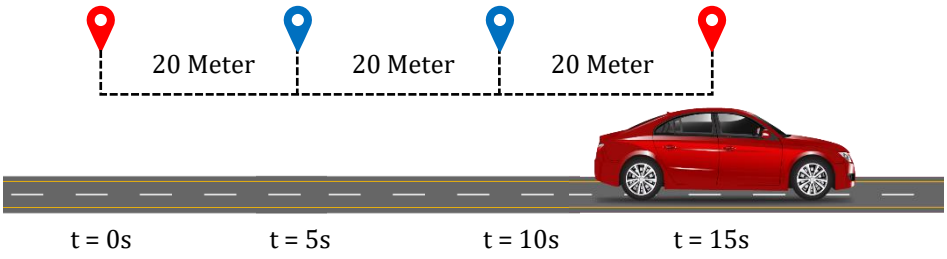
For a moving particle speed can never be zero or negative, it is always positive.



Types of Speed

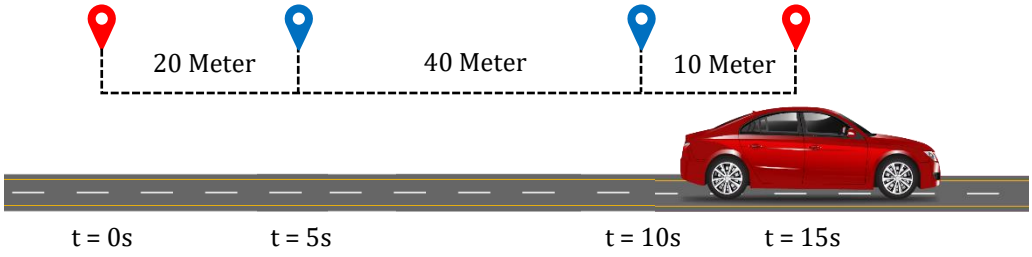
(1) Uniform Speed

Particle covers equal distances in equal interval of time. It is said to be moving with uniform speed.



(2) Non-uniform Speed (Variable Speed)

Particle covers unequal distances in equal intervals of time.



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(3) Average Speed

For a given time interval is defined as the ratio of total distance travelled to total time taken.

$$\text{Average speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}} \quad \text{i.e.} \quad V_{avg} = \frac{\Delta s}{\Delta t}$$

(4) Instantaneous speed

It is the speed of a particle at a particular instant of time.

$$V_{inst.} = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$$

Average speed

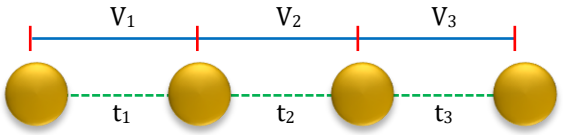
Cases of average speed to Remember

Case I : Particle moves with different uniform speeds v_1, v_2, v_3,.....v_n in different time intervals its average speed over the total time of journey is given as –

$$V_{avg} = \frac{\text{Total Distance covered}}{\text{Total time}} = \frac{s_1 + s_2 + s_3 + ... + s_n}{t_1 + t_2 + t_3 + ... + t_n} = \frac{v_1 t_1 + v_2 t_2 + v_3 t_3 + ...}{t_1 + t_2 + t_3 + ...}$$

If $t_1 = t_2 = t_3 = ...= t_n$ then

$$V_{avg} = \frac{v_1 + v_2 + ... + v_n}{n}$$



For 2 equal intervals of times

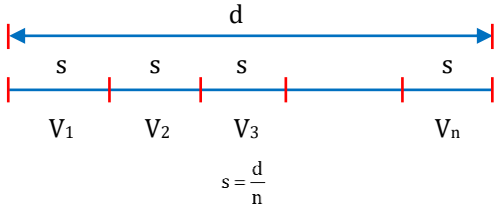
$$V_{avg} = \frac{v_1 + v_2}{2}$$

Case II : Particle describes equal distances with different speeds then the average speed of particle over the total distance will be given as –

If a person moves a certain distance in n equal parts with different speeds.

$$V_{avg} = \frac{d}{\frac{d}{nv_1} + \frac{d}{nv_2} + ... \frac{d}{nv_n}}$$

$$V_{avg} = \frac{n}{\frac{1}{v_1} + \frac{1}{v_2} + ... \frac{1}{v_n}}$$

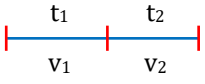


For 2 equal distances.

$$V_{avg} = \frac{2v_1 v_2}{v_1 + v_2}$$

Illustration 10:

A particle moves for 20 sec such that first 10 sec it moves with 36 km/hr and for next 10 sec moves with 54 km/hr. Find average speed.



Solution:

$$v_1 = 36 \times \frac{5}{18} = 10 m/s \quad \text{and} \quad v_2 = 54 \times \frac{5}{18} = 15 m/s$$

$$v_{avg} = \frac{\text{total distance}}{\text{total time}} = \frac{s_1 + s_2}{t_1 + t_2} = \frac{10 \times 10 + 15 \times 10}{20} = \frac{250}{20} = 12.5 m/s = 12.5 \times \frac{18}{5} = 45 km/hr$$

Illustration 11:

A train travels from city A to city B with a constant speed of 10 m/s and return back to city A with a constant speed of 20 m/s. Find its average speed during its entire journey.

Solution:

Let the distance between the two cities A and B = x m.

Time taken by the train to travel from A to B = $\frac{x}{10} = t_1$

Time taken to come back from B to A = $\frac{x}{20} = t_2$

Average speed = $\frac{\text{total distance}}{\text{total time}} = \frac{x+x}{t_1+t_2} = \frac{2x}{\frac{x}{10}+\frac{x}{20}} = \frac{40}{3}\text{m/s}$

Illustration 12:

The displacement of a point moving along a straight line is given by

$s = 4t^2 + 5t - 6$

Here s is in cm and t is in seconds calculate

(i) Initial speed of particle

(ii) Speed at t = 4s

(iii) Average speed in time interval t = 0 to t = 10 sec

Solution:

(i) Speed, $v = \frac{ds}{dt} = 8t + 5$

Initial speed (at t = 0), v = 5 cm/s

(ii) At t = 4s, v = 8(4) + 5 = 37 cm/s

(iii) $S_{(t=0)} = 0 + 0 - 6 = -6$

$S_{(t=10)} = 400+50-6 = 444$

Average speed = $\frac{\text{total distance}}{\text{total time}} = \frac{450}{10} = 45\text{m/s}$

Illustration 13:

A particle covers first one third distance with speed 10m/s and remaining distance with speed 20m/s. Find average speed.

Solution:

$v_{avg} = \frac{d}{t_1+t_2} = \frac{d}{\frac{d}{3\times10}+\frac{2d}{3\times20}} = \frac{3\times10\times20}{2\times10+20} = \frac{600}{40} = 15\text{m/s}$

Velocity

The rate of change of position with time is called velocity.

It is a Vector Quantity.

Dimension : $[M^0L^1T^{-1}]$

Unit: S.I. → m/s

C.G.S. → cm/s

Velocity can be positive, Negative or Zero.

Types of Velocity

(1) Uniform velocity (Constant velocity)

If magnitude as well as direction of its velocity remains same.

This is possible only when it moves in a straight line without reversing its direction.

(2) Non-uniform velocity (variable)

If either magnitude or direction or both varies then velocity changes.

(3) Average velocity

It is defined as the ratio of displacement to time taken by body.

$$\text{Average velocity} = \frac{\text{Net Displacement}}{\text{Time taken}} \Rightarrow \vec{V}_{avg} = \frac{\Delta\vec{r}}{\Delta t} = \frac{\vec{r}_f - \vec{r}_i}{t_f - t_i}$$

Its direction is along the displacement.

Average velocity and Average Speed

Illustration 14:

A particle moves 20 m in 2 sec due east, then it moves 40 m in 3 sec due south. Then find out.

(i) Distance travelled (ii) Displacement (iii) Average speed (iv) Average velocity

Solution:

(i) Distance = 20 + 40 = 60 m

(ii) Displacement = $20\hat{i} - 40\hat{j}$

(iii) Average speed = $\frac{\text{Total distance}}{\text{Total time}} = \frac{60}{5} = 12\text{m/s}$

(iv) Average velocity = $\frac{\text{Net displacement}}{\text{Total time}} = \frac{20\hat{i} - 40\hat{j}}{5} = (4\hat{i} - 8\hat{j})\text{m/s}$

$\vec{v}_{avg} = 4\sqrt{5}$ m/s, E $\tan^{-1}(2)$ S (E to S)

Illustration 15:

A bird flies due north at velocity 20 ms^{-1} for 15 s it rests for 5 s and then flies due south at velocity 24ms^{-1} for 10 s. For the whole trip find the average speed and magnitude of average velocity.

Solution:

Average speed = $\frac{\text{Total Distance}}{\text{Total Time}} = \frac{20\times15 + 24\times10}{15+5+10} = \frac{540}{30} = 18ms^{-1}$

Average velocity = $\frac{\text{Displacement}}{\text{Total Time}} = \frac{(20\times15)\hat{j} + (24\times10)(-\hat{j})}{15+5+10} = \frac{60\hat{j}}{30} = 2\hat{j}$

Magnitude of average velocity = $|2\hat{j}|$ = 2 m/s

Illustration 16:

A car moves with a velocity 2.24 km/h in first minute, with 3.60 km/h in the second minute and with 5.18 km/h in the third minute. Calculate the average velocity in these three minutes.

Solution:

Distance travelled in first minute $s_1 = v_1 \times t_1 = 2.24 \times \frac{1}{60}$ km

Distance travelled in second minute $s_2 = v_2 \times t_2 = 3.60 \times \frac{1}{60}$ km

Distance travelled in third minute $s_3 = v_3 \times t_3 = 5.18 \times \frac{1}{60}$ km

Total distance travelled $s = s_1 + s_2 + s_3 = \frac{2.24}{60} + \frac{3.60}{60} + \frac{5.18}{60} = \frac{11.02}{60}$ km

Total time taken, t = 1 + 1 + 1 = 3 min = $\frac{1}{20}$ h

$\therefore$ average velocity = $\frac{s}{t} = \frac{11.02}{60} \times \frac{20}{1} = 3.67$ km/h

Note:-

(i) Time average velocity

If $v = f(t) \Rightarrow \langle v \rangle = \frac{\int_{t_1}^{t_2} v\,dt}{\int_{t_1}^{t_2} dt}$

(ii) Space average velocity

If $v = f(x) \Rightarrow \langle v \rangle = \frac{\int_{x_1}^{x_2} v\,dx}{\int_{x_1}^{x_2} dx}$

Illustration 17:

If velocity of particle is given by $v = (2t^2+3t+1)$, where 't' is time. Find average velocity for interval $0 \le t \le 2$ sec.

Solution:

$$< v >= \frac{\int_0^2 v\,dt}{\int_0^2 dt} \rightarrow \frac{\left[\frac{2t^3}{3} + \frac{3}{2}t^2 + t\right]_0^2}{2-0} = \frac{20}{3} \text{ m/s}$$

Illustration 18:

If v = sinx, find average velocity for interval $0 \le x \le \frac{\pi}{2}$.

Solution:

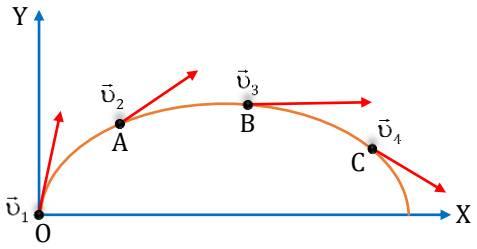
$$< v >= \frac{\int_0^{\frac{\pi}{2}} v\,dx}{\int_0^{\frac{\pi}{2}} dx} = \frac{\int_0^{\pi/2} \sin x\,dx}{\frac{\pi}{2} - 0} = \frac{[-\cos]_0^{\frac{\pi}{2}}}{\frac{\pi}{2}} = \frac{2}{\pi} \text{ m/s}$$

Instantaneous Velocity

It is the velocity of a particle at a particular instant of time.

$$\vec{v}_{avg} = \frac{\Delta \vec{r}}{\Delta t}$$

$$\vec{v}_{inst} = \lim_{\Delta t \to 0}\left(\frac{\Delta \vec{r}}{\Delta t}\right) \Rightarrow \vec{v}_{inst} = \frac{d\vec{r}}{dt}$$



The direction of instantaneous velocity is always tangential to the path followed by the particle.

Important Points

Average speed $\ge$ |Average velocity|

- When particle moves with constant velocity then magnitude of displacement and distance covered by particle is same.
- A particle may have constant speed but variable velocity.

- A particle may have constant speed but variable velocity. Example: When a particle is performing uniform circular motion then for every instant of its circular motion its speed remains constant but velocity changes at every instant.
- When a particle is moving on any path, the magnitude of instantaneous velocity is equal to the instantaneous speed.
- When particle moves with uniform velocity then its instantaneous speed, magnitude of instantaneous velocity, average speed and magnitude of average velocity are all equal.
- When a particle is moving on any path, the magnitude of instantaneous velocity is equal to the instantaneous speed.

Illustration 19:

Position of particle as function of time is given by $\vec{R} = t^2\hat{i} + 2t\hat{j} + (\sin t)\hat{k}$.

Find velocity as function of time.

Solution:

velocity $\vec{v} = \frac{d\vec{r}}{dt} = 2t\,\hat{i} + 2\hat{j} + \cos t\,\hat{k}$

Illustration 20:

A particle is moving along a straight-line OX. At a time t (in seconds) the distance x (in metres) of particle from point O is given by $x = 10 + 6t - 3t^2$. How long would the particle travel before coming to rest?

Solution:

Initial value of x, at t = 0, $x_1 = 10$m

Velocity $v = \frac{dx}{dt} = 6 - 6t$ When v = 0, t = 1s

Final value of x, at t = 1s, $x_2 = 10 + 6 \times 1 - 3(1^2) = 13$ m

Distance travelled = $x_2 - x_1 = 13 - 10 = 3$m

Change in velocity

$\Delta\vec{v} = \vec{v}_f - \vec{v}i$

If $|\vec{v}_f| = |\vec{v}_i| = v$, then

$$|\Delta\vec{v}| = 2v\sin\left(\frac{\theta}{2}\right)$$

where θ is the angle between initial and final velocity

Illustration 21:

A car is moving with speed 10 m/s along north, then it takes 60° left turn and moves with same speed. Find out change in velocity.

Solution:

$\vec{u} = 10(\hat{j})\ m/s$

$\vec{v} = 5\sqrt{3}(-\hat{i}) + 5(+\hat{j})\ m/s$

$\Delta\vec{v} = \vec{v} - \vec{u} = \left(5\sqrt{3}(-\hat{i}) + 5(+\hat{j})\right) - 10(\hat{j}) = -5\sqrt{3}\hat{i} - 5\hat{j}$

$|\Delta\vec{v}| = \sqrt{\left(5\sqrt{3}\right)^2 + 5^2} = 10\,m/s$

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BEGINNER'S BOX-2

1. Air distance between Kota to Jaipur is 260 km and road distance is 320 km. A deluxe bus which moves from Jaipur to Kota takes 8 h while an aeroplane reaches in just 15 min. Find
(i) average speed of bus in km/h
(ii) average velocity of bus in km/h
(iii) average speed of aeroplane in km/h
(iv) average velocity of aeroplane in km/h

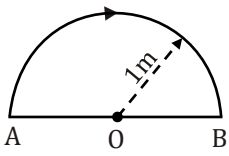
2. A particle moves on a straight line in such way that it covers 1st half distance with speed 3 m/s and next half distance in 2 equal time intervals with speeds 4.5 m/s and 7.5 m/s respectively. Find average speed of the particle.

3. Length of a minute hand of a clock is 4.5 cm. Find the average velocity of the tip of minute's hand between 6 A.M. to 6.30 A.M. & 6 A.M. to 6.30 P.M.

4. A particle of mass 2 kg moves on a circular path with constant speed 10 m/s. Find change in speed and magnitude of change in velocity. When particle completes half revolution.

5. The distance travelled by a particle in time t is given by $s = (2.5\ t^2)$ m. Find (a) the average speed of the particle during time 0 to 5.0s and (b) the instantaneous speed at t = 5.0 s.

6. A particle goes from point A to point B, moving in a semicircle of radius 1m in 1 second. Find the magnitude of its average velocity.



7. Straight distance between a hotel and a railway station is 10 km, but circular route is followed by a taxi covering 23 km in 28 minute. What is average speed and magnitude of average velocity? Are they equal?

Acceleration

Rate of change of velocity is called acceleration.
It is a vector quantity.
Its direction is same as that of change in velocity (not in the direction of the velocity).
Dimension: $[M^0L^1T^{-2}]$
Unit : (S.I.) $\rightarrow$ m/s^2 (C.G.S.) $\rightarrow$ cm/s^2
There are 3 ways to change a velocity (vector)

- Only magnitude change
- Only direction change
- Both direction+ magnitude change

Types of Acceleration

(1) Uniform Acceleration (2) Non-Uniform Acceleration
(3) Average Acceleration (4) Instantaneous acceleration

(1) Uniform Acceleration

A body is said to have uniform acceleration if magnitude and direction of the acceleration remains constant during motion of particle.

(2) Non-Uniform Acceleration

A body is said to have non-uniform acceleration, if either magnitude or direction or both change during motion.

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(3) Average Acceleration

It is the ratio of total change in velocity to the total time taken by the particle

$$\vec{a}_{avg} = \frac{\text{change in velocity}}{\text{time interval}} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$$

Direction of average acceleration is along the change in velocity

(4) Instantaneous Acceleration

It is the acceleration of a particle at a particular instant of time.

$$a_{inst} = \lim_{\Delta t \to 0}\left(\frac{\Delta v}{\Delta t}\right) \quad \Rightarrow \quad a_{inst} = \frac{dv}{dt}$$ When $\vec{v}$ in function of time (t).

i.e. first derivative of velocity is called instantaneous acceleration

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2} \quad \left[\text{As } \vec{v} = \frac{d\vec{r}}{dt}\right]$$

i.e. second derivative of position vector is called instantaneous acceleration

$$a_{inst} = \frac{dv}{dt} \times \frac{dx}{dt} \quad \Rightarrow \quad a_{inst} = v\frac{dv}{dx}$$ When $\vec{v}$ in function of position (x).

Application of Calculus

$$x \xrightarrow[\text{Differentiate w.r.t.}]{\frac{dx}{dt}} v \xrightarrow[\text{Differentiate w.r.t.}]{\frac{dv}{dt}} a$$

$$a \xrightarrow[\text{Integrate w.r.t.}]{\int a\,dt} \Delta v \xrightarrow[\text{Integrate w.r.t.}]{\int v\,dt} \Delta x$$

For v-t relation $\Rightarrow a = \frac{dv}{dt}$ For v-x relation $\Rightarrow a = v\frac{dv}{dx}$

Illustration 22:

The velocity of a particle is given by $v = (2t^2 - 4t + 3)$ m/s where t is time in seconds. Find its acceleration at t = 2 second.

Solution:

Acceleration (a) = $\frac{dv}{dt} = \frac{d}{dt}(2t^2 - 4t + 3) = 4t - 4$

Therefore acceleration at t = 2s is equal to, $a = (4 \times 2) - 4 = 4$ m/s^2

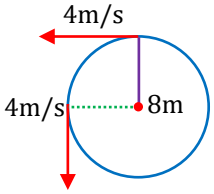
Illustration 23:

A particle is moving along a circular path of radius 8m with uniform speed 4m/s. Find out magnitude of average acceleration when particle completes ¼ half revolution.

Solution:

Average acceleration = $\frac{\text{change in velocity}}{\text{Time}}$

$$\text{time} = \frac{\frac{1}{4}(2\pi r)}{V}$$



$$t = \frac{2\pi \times 8}{4 \times 4} = \pi \text{ s} \qquad \text{so,} \quad a_{avg} = \frac{-4\hat{j} - 4\hat{i}}{\pi} = \frac{4\sqrt{2}}{\pi} \text{ m/s}^2$$

Illustration 24:

The velocity of a body depends on time according to the equation $v = 20 + 0.1t^2$. The body is undergoing what kind of acceleration?

Solution:

Acceleration $a = \frac{dv}{dt} = 0.1 \times 2t = 0.2t$

Which is time dependent i.e. non-uniform acceleration.

Illustration 25:

One particle have velocity $v = \sqrt{x+1}$ then find acceleration of particle?

Solution:

$$\frac{dv}{dx} = \frac{1}{2\sqrt{x+1}} \quad \Rightarrow \quad v\frac{dv}{dx} = \sqrt{x+1}\frac{1}{2\sqrt{x+1}} = \frac{1}{2}$$

Illustration 26:

Position of a particle is given by $x = (6t^2 - t^3)$m, then find out its position when velocity becomes maximum.

Solution:

$$x = 6t^2 - t^3 \quad \Rightarrow \quad v = \frac{dx}{dt} = 12t - 3t^2$$

For maximum velocity:

$$\frac{dv}{dt} = 0 \Rightarrow 12 - 6t = 0 \qquad t = 2 \text{ sec}$$

$\therefore$ Position at t = 2 sec.

$x = 6(2)^2 - (2)^3 = 16$ m

Illustration 27:

The acceleration of a particle moving in a straight line varies with its displacement as, a = 2s+1 velocity of the particle is zero at zero displacement. Find the corresponding velocity - displacement equation.

Solution:

$$a = 2s + 1 \Rightarrow \frac{dv}{dt} = 2s + 1 \Rightarrow \frac{dv}{ds}.\frac{ds}{dt} = 2s + 1 \Rightarrow \frac{dv}{ds}.v = 2s + 1$$

$$\Rightarrow \int_0^v v dv = 2\int_0^s s ds + \int_0^s ds \Rightarrow \left(\frac{v^2}{2}\right)_0^v = 2\left(\frac{s^2}{2}\right)_0^s + [s]_0^s \Rightarrow \frac{v^2}{2} = s^2 + s \Rightarrow v = \sqrt{2s(s+1)}$$

- When particle moves with constant velocity then its acceleration will be zero.
  If particle moves with constant velocity then $v_{avg} = |\vec{v}_{avg}| = v_{inst} = |\vec{v}_{inst}|$
  Acceleration which opposes the motion of body is called retardation.
  Increment or decrement in speed depends on direction of $\vec{v}$ and $\vec{a}$.

Case- I

If $\vec{v}$ and $\vec{a}$ are in same direction then speed increases

(i) $\xrightarrow[\vec{a}=+ve]{\vec{V}=+ve}$ (ii) $\xleftarrow[\vec{a}=-ve]{\vec{V}=-ve}$

Case- II

If $\vec{v}$ and $\vec{a}$ are in opposite direction then speed decreases.

(i) $\xrightarrow[\vec{a}=-ve]{\vec{V}=+ve}$ (ii) $\xleftarrow[\vec{a}=+ve]{\vec{V}=-ve}$

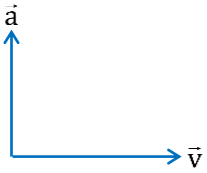
Velocity and Acceleration

Case-I

Angle between $\vec{v}$ & $\vec{a}$ is zero or they are parallel, speed increases

Case-II

Angle between $\vec{v}$ & $\vec{a}$ is 90° or they are orthogonal, speed remains constant



Case-III

Angle between $\vec{v}$ & $\vec{a}$ is 180° or they are anti-parallel, speed decreases

Illustration 28:

For a moving body at any instant of time. Find given statements are true or false?

(i) If the body is not moving, the acceleration is necessarily zero

(ii) If the body is slowing, the retardation is negative

(iii) If the body is slowing, the distance is negative

(iv) If displacement, velocity and acceleration at that are known, we can find the displacement at any given time in future

Solution:

(i) False (ii) False (iii) False (iv) True

Path of Motion Based on Acceleration (To Remember)

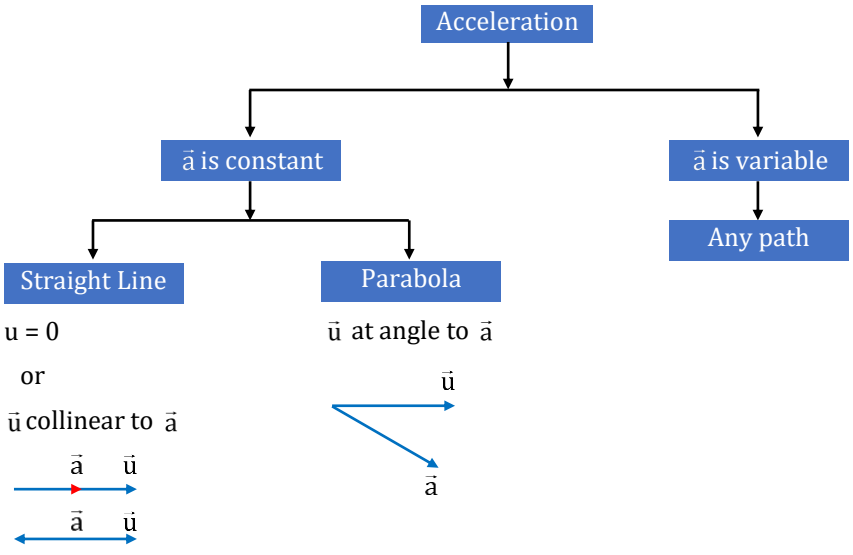


Illustration 29:

The path of a particle moving under the influence of a force fixed in magnitude and direction is (force and velocity at angle θ = 0° or θ = 180°)

Solution:

$\vec{u}$ collinear with $\vec{a}$ i.e. straight line

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BEGINNER'S BOX-3

1. A particle moves on circular path of radius 5 m with constant speed 5 m/s. Find the magnitude of its average acceleration when it completes half revolution.

2. The position of a particle moving on X-axis is given by $x = At^2 + Bt + C$ The numerical values of A, B and C are 7, –2 and 5 respectively and SI units are used. Find

(a) The velocity of the particle at t = 5

(b) The acceleration of the particle at t = 5

(c) The average velocity during the interval t = 0 to t = 5

(d) The average acceleration during the interval t = 0 to t = 5

Equations of Motion

S.No.	Scalar Form	Vector Form
first equation	$v = u + at$	$\vec{v} = \vec{u} + \vec{a}t$
second equation	$s = ut + \frac{1}{2}at^2$	$\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$
third equation	$v^2 = u^2 + 2as$	$\vec{v}\cdot\vec{v} = \vec{u}\cdot\vec{u} + 2\vec{a}\cdot\vec{s}$

Here :

v = final velocity s = displacement

u = initial velocity t = time

a = acceleration = constant

These are valid only when acceleration is constant.

(i) $a = \frac{dv}{dt}$

$\int_v^v dv = \int_0^t a\,dt$

$v - u = at$

$v = u + at$

(ii) $v = \frac{ds}{dt}$

$\int_0^s ds = \int_0^t v\,dt$

$s = ut + \frac{1}{2}at^2$

(iii) $a = v\frac{dv}{ds}$

$\int_u^v v\,dv = \int_0^s a\,ds$

$\left[\frac{v^2}{2}\right]_u^v = a[s]_0^s$

$v^2 = u^2 + 2as$

Some other useful equations

(i) Displacement in the n^{th} second, $s_{n^{th}} = u + \frac{1}{2}a(2n-1)$

(ii) $s = v_{av}t = \frac{(u+v)}{2}t$ (iii) $s = vt - \frac{1}{2}at^2$

Vector form of following equations

(i) $\vec{s}_{n^{th}} = \vec{u} + \frac{1}{2}\vec{a}(2n-1)$ (ii) $\vec{s} = \left(\frac{\vec{u}+\vec{v}}{2}\right)t$ (iii) $\vec{s} = \vec{v}t - \frac{1}{2}\vec{a}t^2$

Steps to follow for solving numerical:

(1) Select origin. Generally, we take starting point as origin if not given.

(2) Select any one direction as positive.

(3) As per direction selected in step 2, write all known variable with sign.

(4) Apply the suitable equations and put variables values with sign.

Key points

When particle starts from rest and moves with constant acceleration then ratio of distance travelled by it in successive equal intervals of time is

1 : 3 : 5 : 7..... (2n – 1)

Known as **Galileo's law of odd numbers**.

Illustration 30:

The velocity of a body moving with a uniform acceleration of $2\ m/sec^2$ is 10 m/sec. Its velocity after an interval of 4 sec is

Solution:

$v = u + at = 10 + 2 \times 4 = 18\ m/sec$

Illustration 31:

The initial velocity of the particle is 10 m/sec and its retardation is $2m/sec^2$. The distance moved by the particle in 5^{th} second of its motion is

Solution:

$$S_n = u - \frac{a}{2}(2n-1) = 10 - \frac{2}{2}(2\times5-1) = 1\ meter$$

Illustration 32:

If a car at rest accelerates uniformly to a speed of 144 km/h in 20 s. Then it covers a distance of

Solution:

Here

v = 144 km/h = 40 m/s

$$v = u + at \Rightarrow 40 = 0 + 20 \times a \Rightarrow a = 2\ m/s^2$$

$$\therefore s = \frac{1}{2}at^2 = \frac{1}{2}\times2\times(20)^2 = 400\ m$$

Illustration 33:

A body starts from rest. What is the ratio of the distance travelled by the body during the 4th and 3rd second

Solution:

$$S_n = u + \frac{a}{2}(2n-1) = \frac{a}{2}(2n-1)$$

because u = 0

Hence $\frac{S_4}{S_3} = \frac{7}{5}$

Illustration 34:

A particle starting from rest travels a distance x in first 2 seconds and a distance y in next two seconds, then

Solution:

If particle starts from rest and moves with constant acceleration then in successive equal interval of time the ratio of distance covered by it will be

1:3:5:7 .....(2n-1)

i.e. ratio of x and y will be a : 3 i.e. $\frac{x}{y} = \frac{1}{3} \Rightarrow y = 3x$

Illustration 35:

Two cars start off a race with velocities 2m/s and 4m/s travel in straight line with uniform accelerations $2m/s^2$ and $1\ m/s^2$ respectively. What is the length of the path if they reach the final point at the same time?

Solution:

Let both particles reach at same position in same time t then from $s = ut + \frac{1}{2}at^2$

For 1st particle : $s = 4(t) + \frac{1}{2}(1)t^2 = 4t + \frac{t^2}{2}$

For 2nd particle : $s = 2(t) + \frac{1}{2}(2)t^2 = 2t + t^2$

Equating above equation we get $4t + \frac{t^2}{2} = 2t + t^2 \Rightarrow t = 4s$

Substituting value of t in above equation $s = 4(4) + \frac{1}{2}(1)(4)^2 = 16 + 8 = 24$ m

Illustration 36:

$x = 8t - t^2$

Find displacement and distance travelled from 0 to 5 sec.?

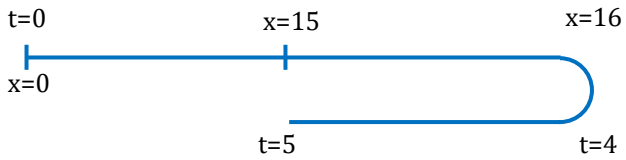
Solution:

0 – 5 sec.

$s = x_f - x_i = 8(5) - 5^2 - (0 - 0)$

displacement s = 15 m

$v = \frac{dx}{dt} = 8 - 2t \Rightarrow v = 2(4 - t) \Rightarrow 0 = 2(4 - t)$



$\therefore$ t = 4 sec. turning point

distance = $|x_4 - x_0| + |x_5 - x_4| = |16 - 0| + |15 - 16| = 16 + 1 = 17$ m

Problems based on Uniformly Accelerated Motion

Illustration 37:

A particle having initial velocity 10 m/s, moves with uniform acceleration. After 2 seconds its velocity becomes 32 m/s. Find distance covered in this duration.

Solution:

u = 10 m/s

a = constant

t = 2s

v = 32 m/s

$s = \left(\frac{u+v}{2}\right)t$

$s = \left(\frac{10+32}{2}\right) \times 2 = 42m$

Average velocity for uniformly accelerated motion

Illustration 38:

A particle moves in a straight line with a uniform acceleration a. Initial velocity of the particle is zero. Find the average velocity of the particle in first 's' distance.

Solution:

$\because s = \frac{1}{2}at^2 \qquad \therefore \frac{s^2}{t^2} = \frac{1}{2}as \qquad$ Average velocity $= \frac{s}{t} = \sqrt{\frac{as}{2}}$

Illustration 39:

A car is moving along a straight road with a uniform acceleration. It passes through two points P and Q separated by a distance with velocity 30 km/h and 40 km/h respectively. The velocity of the car midway between P and Q is?

Solution:

Here, u = 30 km/hr, v = 40 km/hr

Between points A and B

$v_m^2 = 30^2 + 2a(d)$

$2ad = v_m^2 - 900$ ...(i)

Between points B and C

$40^2 = v_m^2 + 2a(d)$

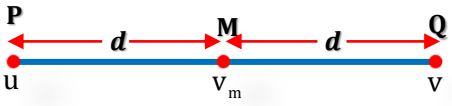
$2ad = 1600 - v_m^2$ ...(ii)

From eq. (1) and (2)

$v_m^2 - 900 = 1600 - v_m^2$

$2v_m^2 = 2500$

$v_m = \sqrt{\frac{2500}{2}}$ km/hr



Stopping Distance

For Vehicles,

When brakes are applied to a moving vehicle, the distance it travels before stopping is called stopping distance.

$a = -a_0$ $u = u_0$

$v = 0$ $s = ?$

$v^2 = u^2 + 2as$

$\Rightarrow 0 = u_0^2 - 2a_0 s \Rightarrow 2a_0 s = u_0^2 \Rightarrow s = \frac{u_0^2}{2a_0}$ [since a is constant] $\Rightarrow s \propto u_0^2$

So, we can say that if u becomes n times then s becomes n^2 times that of previous value.

Stopping Time

$a = -a_0$ $u = u_0$

$v = 0$ $t = ?$

$\Rightarrow v = u + at \Rightarrow 0 = u_0 - a_0 t \Rightarrow a_0 t = u_0 \Rightarrow t = \frac{u_0}{a_0}$ [since a is constant] $\Rightarrow t \propto u_0$

So, we can say that if u becomes n times then t becomes n times that of previous value.

Reaction Time

When a situation demands our immediate action, it takes some time before we really respond. Reaction time is the time a person takes to observe, think and act.

Illustration 40:

A motor car moving with a uniform speed of 20m/sec comes to stop on the application of brakes after travelling a distance of 10m Its acceleration is

Solution:

From $v^2 = u^2 + 2aS \Rightarrow 0 = u^2 + 2aS \Rightarrow a = \frac{-u^2}{2S} = \frac{-(20)^2}{2 \times 10} = -20 m/s^2$

Illustration 41:

A train, travelling at 20 km/hr is approaching a platform. A bird is sitting on a pole on the platform. When the train is at a distance of 2 km from pole, brakes are applied which produce a uniform deceleration in it. At that instant the bird flies towards the train at 60 km/hr and after touching the nearest point on the train flies back to the pole and then flies towards the train and continues repeating itself. Calculate how much distance the bird covers before the train stops?

Solution:

For retardation of train $v^2 = u^2 + 2as \Rightarrow 0 = (20)^2 + 2(a)(2) \Rightarrow a = -100 km/hr^2$

Time required to stop the train v = u + at $\Rightarrow$ 0 = 20 – 100t $\Rightarrow t = \frac{1}{5} hr$

For Bird, speed = $\frac{\text{Distance}}{\text{time}} \Rightarrow s_B = v_B \times t = 60 \times \frac{1}{5} = 12 km$

Illustration 42:

A car moving with a velocity of 10 m/s can be stopped by the application of a constant force F in a distance of 20 m. If the velocity of the car is 30 m/s, it can be stopped by this force in

Solution:

$S \propto u^2$ If u becomes 3 times then S will become 9 times

i.e. 9×20 = 180 m

Illustration 43:

Two trains travelling on the same track are approaching each other with equal speeds of 40 m/s. The drivers of the trains begin to decelerate simultaneously when they are just 2.0 km apart. Assuming the decelerations to be uniform and equal, the value of the deceleration to barely avoid collision should be

Solution:

Both trains will travel a distance of 1 km before to come in rest. In this case by using

$v^2 = u^2 + 2as \Rightarrow 0 = (40)^2 + 2a \times 1000 \Rightarrow a = -0.8 m/s^2$

BEGINNER'S BOX-4

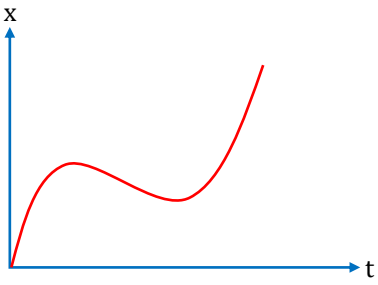
1. A particle starts from rest, moves with constant acceleration for 15s. If it covers s_1 distance in first 5s then distance s_2 in next 10s, then find the relation between s_1 & s_2.
2. The engine of a train passes an electric pole with a velocity 'u' and the last compartment of the train crosses the same pole with a velocity v. Then find the velocity with which the mid-point of the train passes the pole. Assume acceleration to be uniform.
3. A bullet losses 1/n of its velocity in passing through a plank. What is the least number of planks required to stop the bullet? (Assuming constant retardation)
4. A car moving along a straight highway with speed 126 km h^{-1} is brought to a halt within a distance of 200m. What is the retardation of the car (assumed uniform) and how long does it take for the car to stop?
5. A car is moving with speed u. Driver of the car sees red traffic light. His reaction time is t, then find out the distance travelled by the car before coming at rest. Assume uniform retardation 'a' after applying brakes.
6. If a body starts from rest and travels 120cm in the 6^{th} second then what is the acceleration?

Basic Graphs and their Conversions

Mathematical + Graphical approach

$$x \xrightarrow[\text{Slope of x-t graph}]{\frac{dx}{dt}} v \xrightarrow[\text{Slope of v-t graph}]{\frac{dv}{dt}} a \quad : \quad a \xrightarrow[\text{Area of a-t curve with time axis}]{\int a\,dt} \Delta v \quad : \quad v \xrightarrow[\text{Area of v-t curve with time axis}]{\int v\,dt} \Delta x$$

Position-time graph



x-t Graph

- Slope of this graph represents instantaneous velocity.
- Area = Defines No physical Quantity
- Displacement = final position – initial position
- Distance = sum of magnitude of all length one by one.

$$\because \tan\theta = \frac{\text{displacement}}{\text{time}} = \text{velocity}$$

Case-I

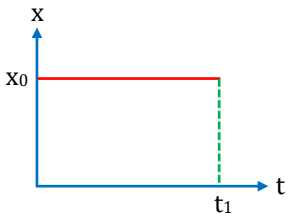
Slope = instantaneous velocity

$\theta = 0$

$\tan\theta = \tan 0 = 0$

velocity = 0

i.e. body is at rest.



Case-II

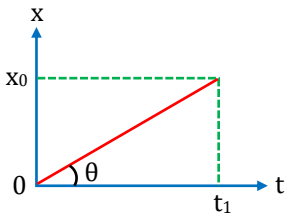
Slope = instantaneous velocity

θ = constant

$\tan\theta$ = constant

velocity = constant

i.e. the body is in uniform motion



Case-III

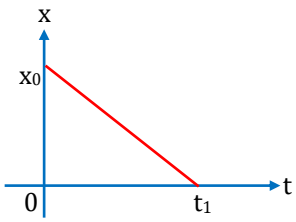
Slope = instantaneous velocity

$\theta > 90$

$\tan\theta$ = –ve

velocity = –ve but constant

i.e. uniform motion



Case-IV

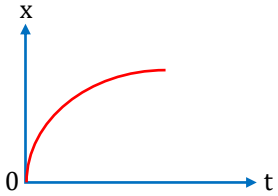
When particle starts from rest and moves with constant acceleration

θ is decreasing with time

$\therefore \tan\theta$ is decreasing with time

$\therefore$ velocity is decreasing with time

i.e. non uniform motion

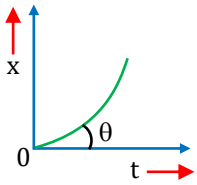


θ is increasing with time

$\therefore \tan\theta$ is increasing with time

$\therefore$ velocity is increasing with time

i.e. non-uniform motion

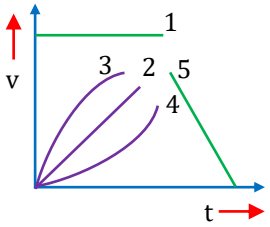


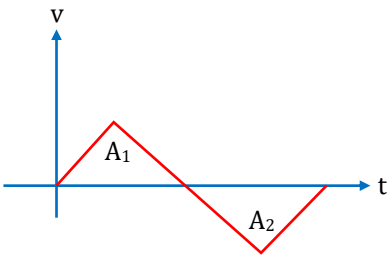
- Area of x-t graph = $\int xdt$ = No physical significance

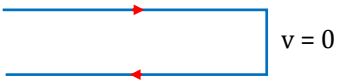
Velocity-time graph

Slope of this graph represents acceleration.

$\because \tan\theta = \frac{\text{velocity}}{\text{time}} = \text{acceleration}$







Area under the v-t curve gives

|displacement| = $A_1 - A_2$

Distance = $A_1 + A_2$

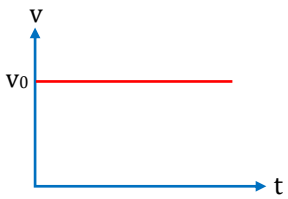
Case-I

$\theta = 0°$

$\tan\theta = \tan 0° = 0$

acceleration = 0

i.e. v = constant or uniform motion



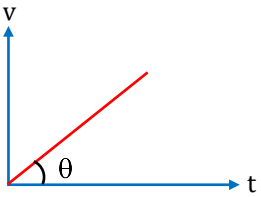
Case-II

θ = constant

tan θ = constant

acceleration = constant

i.e. uniformly accelerated motion



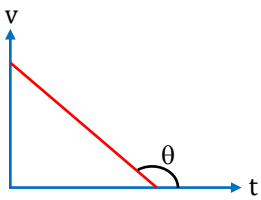
Case-III

$\theta > 90°$

$\tan\theta$ = –ve

acceleration = –ve but constant

i.e. constant or uniform retardation is acting on the body



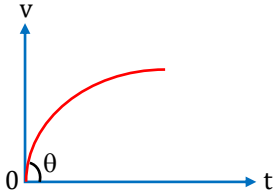
Case-IV

θ is decreasing with time

$\therefore \tan\theta$ is decreasing with time

$\therefore$ acceleration is decreasing with time

i.e. acceleration goes on decreasing with time but it is not retardation

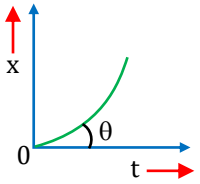


θ is increasing with time

$\therefore \tan\theta$ is increasing with time

$\therefore$ acceleration is increasing with time

i.e. acceleration goes on increasing with time



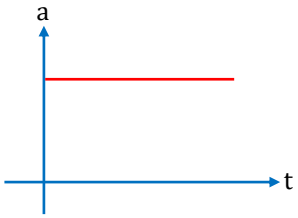
- Area of v-t graph = $\int vdt$ = displacement = change in position

Acceleration-time graph

Slope of a-t graph defines nothing

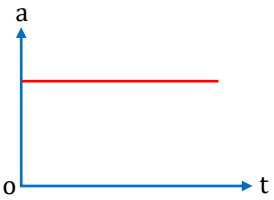
But Area of a-t graph gives change in velocity

$\therefore$ Area of a–t graph = $\int a\,dt = \int dv = v_2 - v_1$ = change in velocity



- Area $\neq$ final velocity

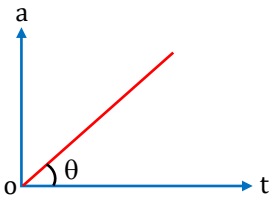
Case-I



$a \propto t^0$

i.e. uniform or constant acceleration

Case-II

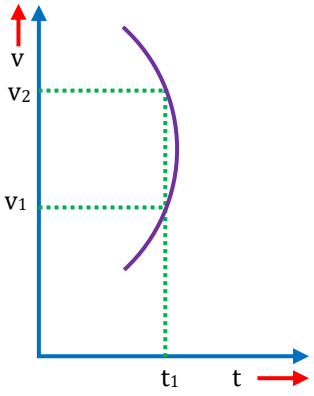


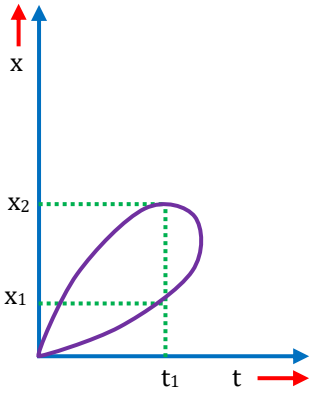
i.e. uniformly increasing acceleration.

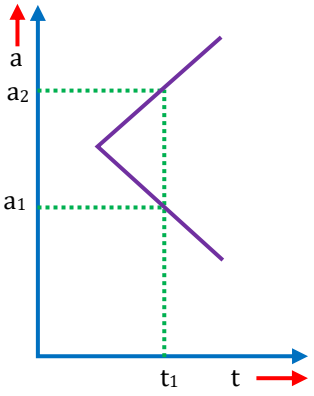
Key Points

Following graphs do not exist in practice :

Case-I



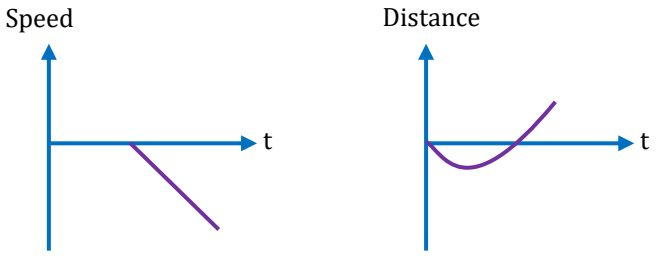




Explanation:

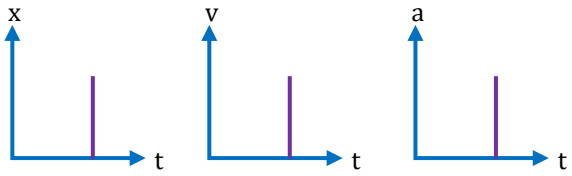
In practice, at any instant body can not have two velocities or displacements or accelerations simultaneously.

Case-II



Explanation:
Speed or distance can never be negative.

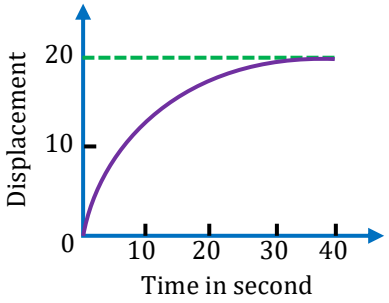
Case-III



Explanation : It is not possible to change any quantity without consuming time i.e. time can't be constant.

Illustration 44:

The displacement of a particle as a function of time is shown in the figure. The figure shows that velocity is

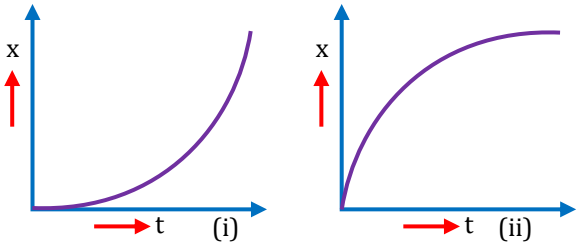


Solution:

The slope of displacement-time graph goes on decreasing, it means the velocity is decreasing i.e. It's motion is retarded and finally slope becomes zero i.e. particle stops.

Illustration 45:

Figures (i) and (ii) below show the displacement-time graphs of two particles moving along the x-axis. We can say that

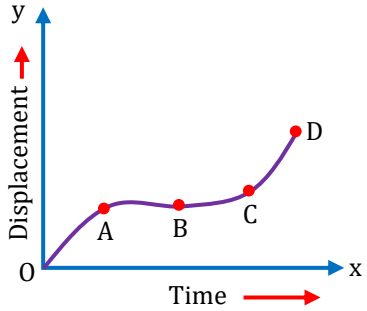


Solution:

Particle (i) is having a uniformly accelerated motion while particle (ii) is having a uniformly retarded motion.

Illustration 46:

The graph between the displacement x and time t for a particle moving in a straight line is shown in figure. During the interval OA, AB, BC and CD, the acceleration of the particle is



Solution:

Region OA shows that graph bending toward time axis i.e. acceleration is negative.

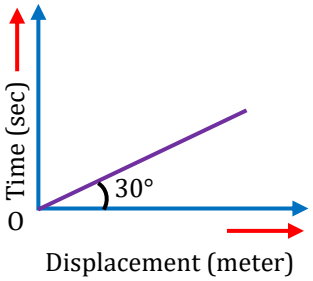
Region AB shows that graph is parallel to time axis i.e. velocity is zero. Hence acceleration is zero.

Region BC shows that graph is bending towards displacement axis i.e. acceleration is positive.

Region CD shows that graph having constant slope i.e. velocity is constant. Hence acceleration is zero.

Illustration 47:

From the following displacement-time graph find out the velocity of a moving body



Solution:

Slope of the x-t graph gives instantaneous velocity but here t-x graph,

$\upsilon = \tan 30^\circ = \frac{1}{\sqrt{3}}$ m/s.

But it is wrong because formula $v = \tan\theta$ is valid when angle is measured with time axis.

Here angle is taken from displacement axis. So angle from time axis = 90° – 30° = 60°

Now $\upsilon = \tan 60^\circ = \sqrt{3}$

Graphical Problems in Horizontal Motion

Shortcut to Remember

If a car, starts from rest moves with constant acceleration α for some time retards uniformly at rate β and finally comes at rest total time of motion is T

Then:

$v_{max} = \left(\frac{\alpha\beta}{\alpha+\beta}\right)T$

$s = \frac{1}{2}\left(\frac{\alpha\beta}{\alpha+\beta}\right)T^2$

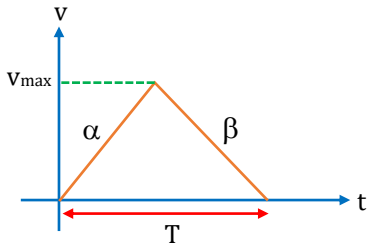


Illustration 48:

A particle is moving along x-axis. Its position is given by $x = -2t + 3t^2$. Draw its :-

(i) v-t graph

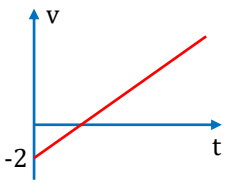
(ii) a-t graph

Solution:

$x = -2t + 3t^2$

$v = -2 + 6t$

$a = 6$



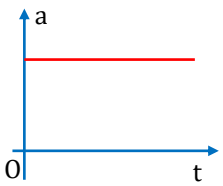
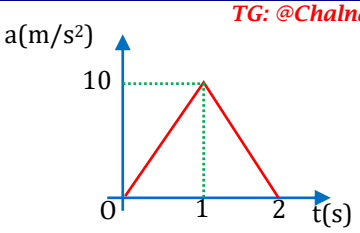


Illustration 49:

A particle have initial velocity 10 m/s. Its acceleration time graph is shown. Find out its velocity at t = 2 sec.



Solution:

$$\Delta V = V_f - V_i = \text{Area} = \frac{1}{2}\times 2\times 10$$

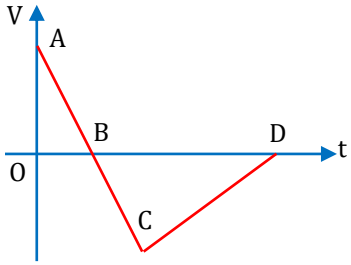
$$V_f - 10 = 10$$

$$V_f = 20 m/s$$

Illustration 50:

For a given velocity time graph, find out :

(i) In which duration acceleration is Positive, Negative and Zero.



Solution:

AB $\rightarrow$ Negative

BC $\rightarrow$ Negative

CD $\rightarrow$ Positive

Illustration 51:

Velocity-time graph for a particle moving in a straight line is given. Calculate the displacement of the particle and distance travelled in first two sec.

Solution:

(i) Displacement = area $= \left(\frac{1}{2}\times 6\times 1\right) + \left[\frac{1}{2}\times 1\times(-6)\right] = 0$

(ii) Distance $= \left(\frac{1}{2}\times 6\times 1\right) + \left(\frac{1}{2}\times 6\times 1\right) = 6m$

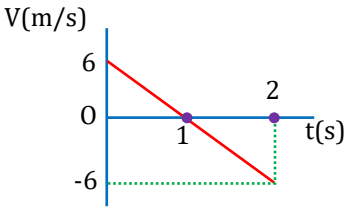
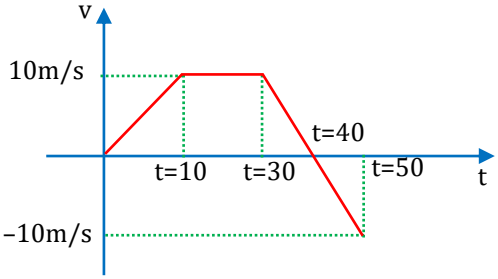


Illustration 52:

Find :

(i) average velocity from t = 0 to t = 30

(ii) average speed from t = 10 to t = 40



Solution:

Average velocity from t = 0 to t = 30

$$< v >= \frac{\Delta x}{\Delta t} = \frac{50+200}{30} = \frac{25}{3} m/s$$

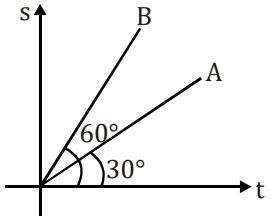
Average speed from t = 10 to t = 40

$$\frac{\text{distance}}{\text{time}} = \frac{200+50}{30} = \frac{25}{3} m/s$$

because there is no turning point.

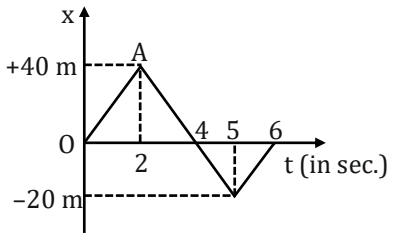
BEGINNER'S BOX-5

1. s-t graph of two particles A and B are shown in fig. Find the ratio of velocity of A to velocity of B.



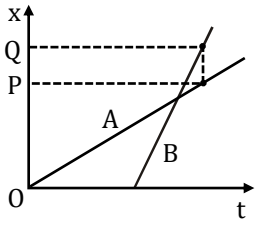
2. Position-time graph of a particle in motion is shown in fig. Calculate –

(i) Total distance covered
(ii) Displacement
(iii) Average speed
(iv) Average velocity.



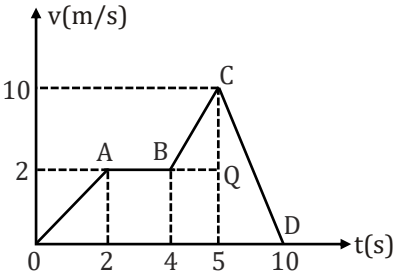
3. The position-time (x-t) graphs for two children A and B returning from their school O to their homes P and Q respectively are shown in fig. Choose correct entries in the brackets below :

(a) (A/B) lives closer to the school than (B/A)
(b) (A/B) starts from the school earlier than (B/A)
(c) (A/B) walks faster than (B/A)
(d) A and B reach home at the (same / different) time
(e) (A/B) overtakes (B/A) on the road (once/ twice).



4. A particle moves on straight line according to the velocity–time graph shown in fig. Calculate –

(i) Total distance covered
(ii) Average speed
(iii) In which part of the graph the acceleration is maximum and also find its value.
(iv) Retardation



5. A body starts from rest and moves with a uniform acceleration of 10 ms^{-2} for 5 seconds. During the next 10 seconds it moves with uniform velocity. Find the total distance travelled by the body (Using graphical analysis).

Motion under Gravity-Vertical Projection from Ground

Motion Under Gravity

Acceleration produced in the body by the force of gravity, is called acceleration due to gravity. It is represented by the symbol g.

Value of g = 9.8 m/s^2 or g = 980 cm/s^2 or g=32 ft/s^2

In the absence of air, it is found that all bodies (irrespective of the size, weight or composition) fall with the same acceleration near the surface of the earth. This motion of a body falling towards the earth from a small altitude (h << earth's radius) is called motion under gravity. Free fall means acceleration of body is equal to acceleration due to gravity.

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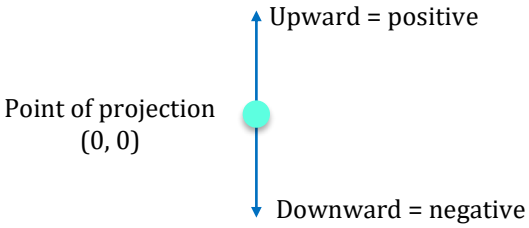
Sign Conventions

Negative and positive sign are matter of our choice, so we can select any direction as positive and opposite side as negative.



Vertical Projection from ground (Body is Projected Vertically Upward)

Equations of motion : Taking initial position as origin and direction of motion (i.e. vertically up) as positive and (vertical downward) as negative.

u (initial velocity) = +u
a (acceleration) = -g
at time t,
Final velocity = v
height above ground = h

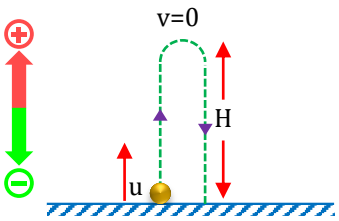
Positive / Negative directions are a matter of choice. You may take another choice.

$\therefore$ $v = u - gt$ ...(i)

$h = ut - \frac{1}{2}gt^2$ ...(ii)

$v^2 = u^2 - 2gh$ ...(iii)

$h_{n^{th}} = u - \frac{g}{2}(2n-1)$



Maximum height (H)

Final velocity (v) = 0 so,
From above equation (iii)

$0 = u^2 - 2gH \quad \Rightarrow \quad H = \frac{u^2}{2g}$

Total time of flight(T)

at maximum height v = 0
So from above equation (i) u = g t
it is called time of ascent (t_1) = u/g
In case of motion under gravity, time taken to go up is equal to the time taken to fall down through the same distance. Time of descent (t_2) = time of ascent (t_1) = u/g
$\therefore$ Total time of flight T = $t_1 + t_2$

$T = \frac{2u}{g}$

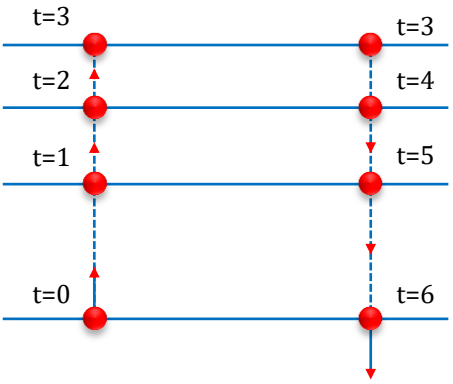


Illustration 53:

A ball is projected vertically upward with a speed of 50 m/s.
Find

(a) the maximum height,
(b) the time to reach the maximum height,
(c) the speed at half the maximum height.
(d) time of flight
(e) velocity after t = 1 and t = 6 s
(f) height from ground at t = 1 and t = 5 sec
Take g = 10m/s^2

Solution:

(a) Initial velocity of ball (u)=50m/s, acceleration of ball =−g,
final velocity at the highest point (v)=0
So applying the 3rd equation of motion we get:

$v^2=u^2-2gh_{max} \Rightarrow 0=50^2-2\times10\times h_{max} \Rightarrow h_{max}=\frac{2500}{20}=125\text{ m}$

(b) Let the time required to reach max height be t. Then applying 1st equation of motion we get:
$v=u-gt \Rightarrow 0=50-10t \Rightarrow t=5s$

(c) Let speed at half of max height be V then:

$V^2=50^2-2g\frac{125}{2} \Rightarrow V^2=2500-1250=1250 \Rightarrow V=\sqrt{1250}=35.35\text{m/s}$

(d) $T=\frac{2u}{g}=\frac{50\times2}{10}=10\text{sec}$

(e) velocity after one second

v = u – gt

v = 50-10×1 = 40 m/s

velocity after 6 sec

v= 50-10×6 = –10m/s or 10 m/s downward

(f) Height after one second from ground

$h_1=50-\frac{1}{2}\times10=45\text{m}$

height after 6 sec from ground

$h_6=50\times6-\frac{1}{2}\times10\times36=120\text{m}$

Illustration 54:

A ball is projected upwards from the foot of a tower. The ball crosses the top of the tower twice after an interval of 6s and the ball reaches the ground after 12s. The height of the tower is ($g=10m/s^2$):

Solution:

$T=\frac{2u}{g}=12$

2u = 120

u = 60 m/s

$h = 60\times3-\frac{1}{2}\times10\times9$

=180-45=135 m

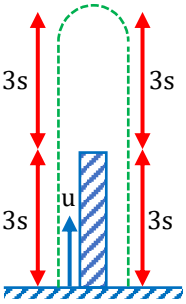


Illustration 55:

A body A is projected upwards with a velocity of 98m/s. The second body B is projected upwards with the same initial velocity but after 4 sec. Both the bodies will meet after

Solution:

Let t be the time of flight of the first body after meeting, then (t-4)sec will be the time of flight of the second body. Since $h_1 = h_2$

$\therefore 98t-\frac{1}{2}gt^2=98(t-4)-\frac{1}{2}g(t-4)^2$

On solving, we get t = 12seconds

Illustration 56:

When a ball is thrown up vertically with velocity V_0, it reaches a maximum height of 'h'. If one wishes to triple the maximum height then the ball should be thrown with velocity

Solution:

$H_{max} \propto u^2 \qquad \therefore u \propto \sqrt{H_{max}}$

i.e. to triple the maximum height, ball should be thrown with velocity $\sqrt{3}u$

Illustration 57:

With what velocity a ball be projected vertically so that the distance covered by it in 5th second is twice the distance it covers in its 6th second (g=10m/s²)

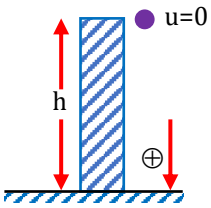
Solution:

$h_{n^{th}} = u - \frac{g}{2}(2n-1)$; $h_{5^{th}} = u - \frac{10}{2}(2\times5-1) = u-45$; $h_{6^{th}} = u - \frac{10}{2}(2\times6-1) = u-55$

Given $h_{5^{th}} = 2\times h_{6^{th}}$. By solving we get u = 65 m/s

Motion under Gravity-Dropping from height

Body dropped from some height (initial velocity zero)



Equations of Motion

Taking initial position as origin and direction of motion (i.e., downward direction) as a positive, here we have

u = 0 [As body starts from rest]

a = +g [As acceleration is in the direction of motion]

at time t,

final velocity = v

height below dropping point = h

$v = gt$ ...(i) $\qquad h = \frac{1}{2}gt^2$ ...(ii) $\qquad v^2 = 2gh$ ...(iii)

Results

from equations (i) (ii) and (iii)

- Time to reach bottom(T)

$T = \sqrt{\frac{2H}{g}}$

- Maximum speed (at bottom)

$v = \sqrt{2gH}$

Illustration 58:

A ball is released from a height and it reaches the ground in 3 s. If g = 9.8 ms⁻², find

(a) the height from where the ball was released

(b) the velocity with which the ball will strike the ground.

(c) at t = 2s, velocity and height from the surface

Solution:

(a) $s = ut + \frac{1}{2}at^2 = \frac{1}{2}\times9.8\times3^2 = 44.1m$

(b) $v = u + at = 0 + (9.8)(3) = 29.4$ m/s

(c) $v = gt = 9.8\times2 = 19.6$m/s ; $h = \frac{1}{2}\times9.8\times2^2 = 19.6m$

Key Points

- As $h = (1/2)gt^2$, i.e., $h \propto t^2$, distance covered in time t, 2t, 3t, etc., will be in the ratio of $1^2 : 2^2 : 3^2$, i.e., square of consecutive integers. (in case of free fall, from rest)
- A particle at rest, is dropped vertically from a height. The time taken by it to fall through successive distance of 1m each will then be in the ratio of the difference in the square roots of the integers i.e. $\sqrt{1},(\sqrt{2}-\sqrt{1}),(\sqrt{3}-\sqrt{2}),(\sqrt{4}-\sqrt{3}),.....$
- The motion is independent of the mass of body, as mass is not involved in any equation of motion. It is due to this reason that a heavy and light body when released from the same height, reach the ground simultaneously and with same velocity i.e., $t=\sqrt{(2h/g)}$ and $v=\sqrt{2gh}$.
- The distance covered in the n^{th} second, $h_n=\frac{1}{2}g(2n-1)$

  So distance covered in 1^{st}, 2^{nd}, 3^{rd} second, etc., will be in the ratio of 1 : 3 : 5, i.e., odd integers only.

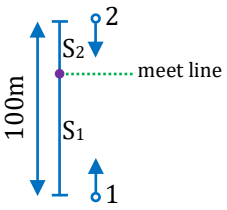
Illustration 59:

A particle is dropped from height 100 m and another particle is projected vertically up with velocity 50 m/s from the ground along the same line. Find out the height where two particle will meet? (take g = 10 m/s²)

Solution:

$S_1=ut-\frac{1}{2}gt^2$ ...(i)

$S_2=\frac{1}{2}gt^2$ ...(ii) $S_1 + S_2 = 100$ m



From eq. (i) and (ii) $S_2=\frac{1}{2}gt^2=\frac{1}{2}\times 9.8\times 2^2=19.62m$ (distance from top)

We get $S_1 + S_2 = ut$ Height (s_1) = 100-19.62=80.38m

ut = 100 $\Rightarrow$ 50t = 100 $\Rightarrow$ t = 2s

Illustration 60:

A body falls freely from rest. It covers as much distance in the last second of its motion as covered in the first three seconds. The body has fallen for a time of

Solution:

$\frac{1}{2}g(3)^2=\frac{g}{2}(2n-1)\Rightarrow n=5\,s$

Illustration 61:

A body is released from the top of a tower of height h. It takes t sec to reach the ground. Where will be the ball after time t/2 sec

Solution:

Let the body after time t/2 be at x from the top, then

$x=\frac{1}{2}g\frac{t^2}{4}=\frac{gt^2}{8}$ ...(i) $h=\frac{1}{2}gt^2$ ...(ii)

Eliminate t from (i) and (ii), we get $x=\frac{h}{4}$

$\therefore$ Height of the body from the ground $=h-\frac{h}{4}=\frac{3h}{4}$

Illustration 62:

A body is released from a great height and falls freely towards the earth. Another body is released from the same height exactly one second later. The separation between the two bodies, two seconds after the release of the second body is

Solution:

The separation between the two bodies, two seconds after the release of second body

$$=\frac{1}{2}\times 9.8[(3)^2-(2)^2]=24.5\,\text{m}$$

Illustration 63:

A body falls from a height h = 200m. The ratio of distance travelled in each 2 sec during t = 0 to t = 6 second of the journey is

Solution:

$S_n \propto (2n-1)$ In equal time interval of 2 seconds

Ratio of distance = 1 : 3 : 5

Motion under Gravity-Vertical Projection from Height

Vertical Projection from Height

CASE 1 : Downward Projection

Equations of motion:

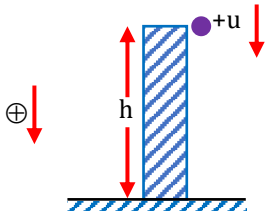
Taking initial position as origin and direction of motion (i.e., downward direction) as a positive, we have

$v = u + gt$ ...(i)

$h = ut + \frac{1}{2}gt^2$ ...(ii)

$v^2 = u^2 + 2gh$ ...(iii)

$h_n = u + \frac{g}{2}(2n-1) \Rightarrow v_{max} = \sqrt{u^2+2gh}$



CASE 2: Upward Projection

Equations of motion :

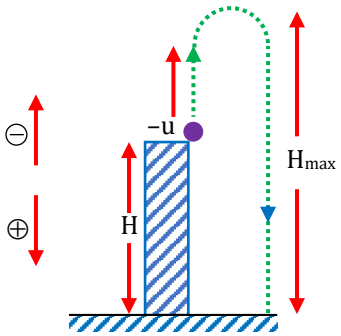
Taking initial position as origin and direction of motion (i.e., upward direction) as negative, here we have

$v = -u + gt$ ...(i)

$h = -ut + \frac{1}{2}gt^2$ ...(ii)

$v^2 = u^2 + 2gh$ ...(iii)

$h_{n^{th}} = -u + \frac{g}{2}(2n-1)$



- Maximum height attained by the body

  $H_{max} = H + \frac{u^2}{2g}$

- Distance travelled by the body = $H + \frac{u^2}{g}$

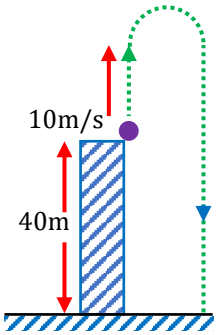
- Time taken by the body to reach the ground

  $H = -ut + \frac{1}{2}gt^2 \Rightarrow \frac{1}{2}gt^2 - ut - H = 0 \Rightarrow gt^2 - 2ut - 2H = 0$

  After solving this equation, we get the result.

Illustration 64:

A ball is thrown upwards from the top of a tower 40m high with a velocity of 10 m/s, find the time when it strikes the ground ($g = 10\ m/s^2$)



Solution:

In the problem $u = +10\ m/s$, $a = -10\ m/s^2$ and $s = -40m$

Substituting in $s = ut + \frac{1}{2}at^2$

$-40 = 10t - 5t^2$ or $5t^2 - 10t - 40 = 0$ or $t^2 - 2t - 8 = 0$

Solving this we have $t = 4$ s and -2s. Taking the positive value $t = 4s$

Illustration 65:

A pebble is thrown vertically upwards from a bridge with an initial velocity of 4.9 m/s. It strikes the water after 2s. If acceleration due to gravity is $9.8m/s^2$ (a) what is the height of the bridge? (b) with what velocity does the pebble strike the water?

Solution:

(a) Let height of the bridge be h then

$$h = -4.9 \times 2 + \frac{1}{2} \times 9.8 \times (2)^2$$

$h = 9.8$ m

(b) velocity with which the ball strikes the water surface

$v = u + at$

$v = -4.9 + 9.8 \times 2 = 14.7$ m/s

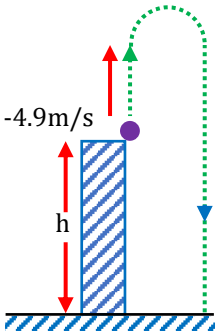


Illustration 66:

A man in a balloon rising vertically with an acceleration of $4.9\,m/sec^2$ releases a ball 2 sec after the balloon is let go from the ground. The greatest height above the ground reached by the ball is $(g = 9.8\,m/sec^2)$

Solution:

Height travelled by ball (with balloon) in 2 sec

$$h_1 = \frac{1}{2}a\,t^2 = \frac{1}{2} \times 4.9 \times 2^2 = 9.8\,m$$

Velocity of the balloon after 2 sec

$$v = a\,t = 4.9 \times 2 = 9.8\,m/s$$

Now if the ball is released from the balloon then it acquire same velocity in upward direction.

Let it move up to maximum height h_2

$$v^2 = u^2 - 2gh_2 \Rightarrow 0 = (9.8)^2 - 2 \times (9.8) \times h_2$$

$$\therefore h_2 = 4.9m$$

Greatest height above the ground reached by the ball $= h_1 + h_2 = 9.8 + 4.9 = 14.7\,m$

Illustration 67:

Three particles A, B and C are thrown from the top of a tower with the same speed. A is thrown up, B is thrown down and C is horizontally. They hit the ground with speeds V_A, V_B and V_C respectively.

Solution:

$v^2 = u^2 + 2gh \Rightarrow v = \sqrt{u^2 + 2gh}$

So, for all the cases velocity will be equal.

Effect of Air Resistance

If 'a' is retardation due to air resistance,

Case-1

For ascending motion(upward motion)

net acceleration a_{Net} = g + a (downwards)

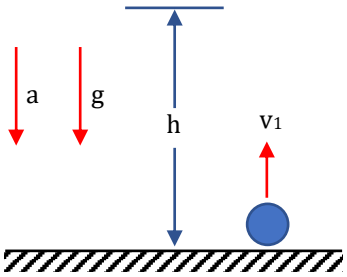
If maximum height attained by the particle is 'h' then

Time to go up,

$t_1 = \sqrt{\frac{2h}{g+a}}$

Velocity of projection,

$v_1 = \sqrt{2(g+a)h}$



Case-2

For descending motion (downward motion)

net acceleration a_{Net} = g - a (downwards)

If body is released from maximum height 'h' then

Time to go up,

$t_2 = \sqrt{\frac{2h}{g-a}}$

Velocity on reaching ground,

$v_2 = \sqrt{2(g-a)h}$

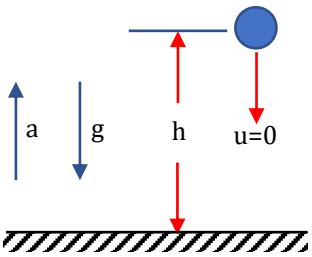


Illustration 68:

A body is thrown vertically upwards. If air resistance is to be taken into account, then the time during which the body rises then the time of fall

Solution:

For ascending motion $t_1 = \sqrt{\frac{2h}{g+a}}$

For descending motion $t_2 = \sqrt{\frac{2h}{g-a}}$

$t_2 > t_1$

$\therefore$ time during the body rises is less than the time of fall.

BEGINNER'S BOX-6

1. A particle is projected vertically upwards from ground with velocity 10 m/s. Find the time taken by it to reach at the highest point?

2. A particle is projected vertically up from the top of a tower with velocity 10 m/s. It reaches the ground in 5s. Find–

(a) Height of tower.

(b) Striking velocity of particle at ground

(c) Distance traversed by particle.

(d) Average speed & average velocity of particle.

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3. A balloon starts rising from the ground with an acceleration of 1.25 m/s². A stone is released from the balloon after 10s. Determine
(1) maximum height of stone from ground (2) time taken by stone to reach the ground

4. A rocket is fired vertically up from the ground with an acceleration 10 m/s². If its fuel is finished after 1 minute then calculate –
(a) Maximum velocity attained by rocket in ascending motion.
(b) Height attained by rocket before fuel is finished.
(c) Time taken by the rocket in the whole motion.
(d) Maximum height attained by rocket.

5. A particle is dropped from the top of a tower. During its motion it covers $\frac{9}{25}$ part of height of tower in the last 1 second Then find the height of tower.

6. A particle is dropped from the top of a tower. It covers 40 m in last 2s. Find the height of the tower.

7. A player throws a ball upwards with an initial speed of 29.4 m/s.
(a) What is the direction of acceleration during the upward motion of the ball?
(b) What are the velocity and acceleration of the ball at the highest point of its motion?
(c) Choose x = 0 m and t = 0 s to be the location and time of the ball at its highest point, vertically downward direction to be the positive direction of x-axis and give the signs of position, velocity and acceleration of the ball during its upward and downward motion.
(d) To what height does the ball rise and how long does the ball take to return to the player's hands? (Take g = 9.8 m/s² and neglect air resistance).

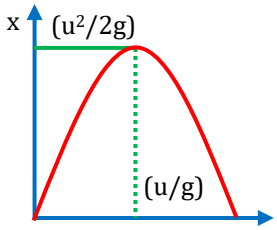
8. A particle is dropped from the top of a tower. The distance covered by it in the last one second is equal to that covered by it in the first three seconds. Find the height of the tower.

9. Water drops are falling in regular intervals of time from top of a tower of height 9 m. If 4th drop begins to fall when 1st drop reaches the ground, find the positions of 2nd & 3rd drops from the top of the tower.

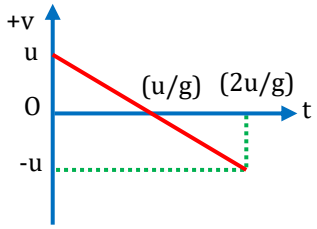
Graphical Problems in Vertical Motion

1. A body is projected vertically upwards then

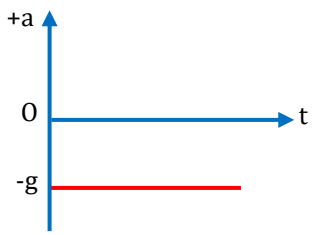
(a) Graphs of displacement with respect to time :



(b) Graph of velocity with respect to time



(c) Graph of acceleration with respect to time



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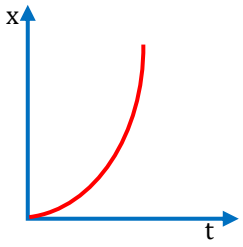
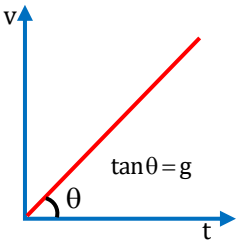
2. A body dropped from some height

(a) Graphs of displacement with respect to time:



(b) Graph of velocity with respect to time



(c) Graph of acceleration with respect to time

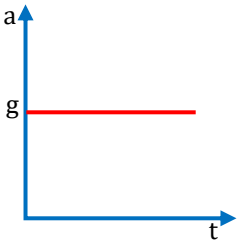


Illustration 69:

A particle is thrown vertically upward with 30 m/s, reaches on maximum height and come on ground after some time. Consider vertically upward direction positive and point of projection as origin. (if g=10 m/sec^2)

(i) Acceleration time graph

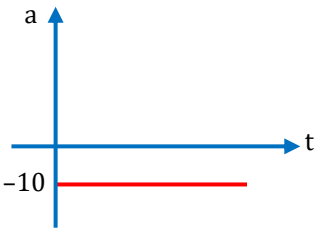
(ii) Velocity time graph

(iii) Speed time graph

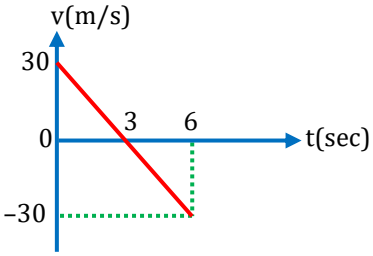
(iv) Displacement time graph

Solution:

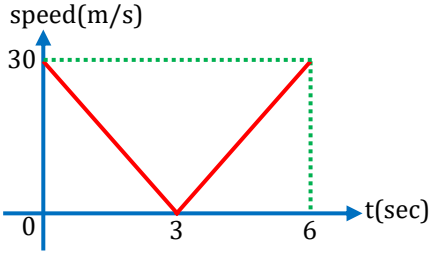
(i) Acceleration time graph



(ii) Velocity time graph



(iii) Speed time graph



(iv) Displacement time graph

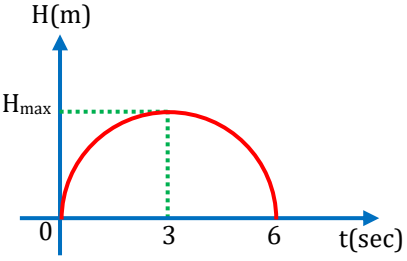
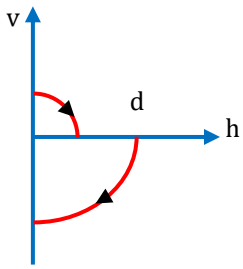


Illustration 70:

A ball is dropped vertically from a height d above the ground. It hits the ground and bounces up vertically to a height d/2. Neglecting subsequent motion and air resistance, its velocity v varies with the height h above the ground is

Solution:

For the given condition initial height h = d and velocity of the ball is zero. When the ball moves downward its velocity increases and it will be maximum when the ball hits the ground & just after the collision it becomes half and in opposite direction. As the ball moves upward its velocity again decreases and becomes zero at height d/2.